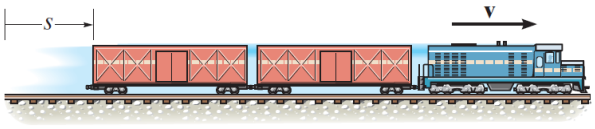
Problem 12-21

A freight train travels at $v = 60(1 - e^{-t})$ ft/s, where t is the elapsed time in seconds. Determine the distance traveled in three seconds, and the acceleration at this time.



Prob. 12–21

Solution

The acceleration can be obtained by differentiating the velocity.

$$\begin{aligned} a &= \frac{dv}{dt} \\ &= \frac{d}{dt}(60 - 60e^{-t}) \\ &= \frac{d}{dt}(60) - \frac{d}{dt}(60e^{-t}) \\ &= (0) - (60e^{-t}) \cdot \frac{d}{dt}(-t) \\ &= 0 - 60e^{-t} \cdot (-1) \\ &= 60e^{-t} \end{aligned}$$

Therefore, after 3 seconds the acceleration is

$$a(3) = 60e^{-3}\ \frac{\text{ft}}{\text{s}^2} \approx 2.99\ \frac{\text{ft}}{\text{s}^2}.$$

The velocity is related to position by

$$v = \frac{ds}{dt} = 60(1 - e^{-t}).$$

Integrate both sides with respect to t to get $s(t)$.

$$s(t) = \int (60 - 60e^{-t})\, dt = 60t + 60e^{-t} + C$$

Assume that $s = 0$ when $t = 0$ in order to determine C.

$$s(0) = 60 + C = 0 \quad \rightarrow \quad C = -60$$

The train's position is then

$$s(t) = 60t + 60e^{-t} - 60.$$

After 3 seconds it is $s(3) = 60(3) + 60e^{-3} - 60 \approx 123$ ft.